Validation of Inverse Dynamics codes for a 3 Axis SCARA manipulator

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Tools used: *Self Developed MATLAB code and multi-body dynamics simulation of 3-axis SCARA in Altair Hyperworks Motionview*

3 Axis SCARA Manipulator

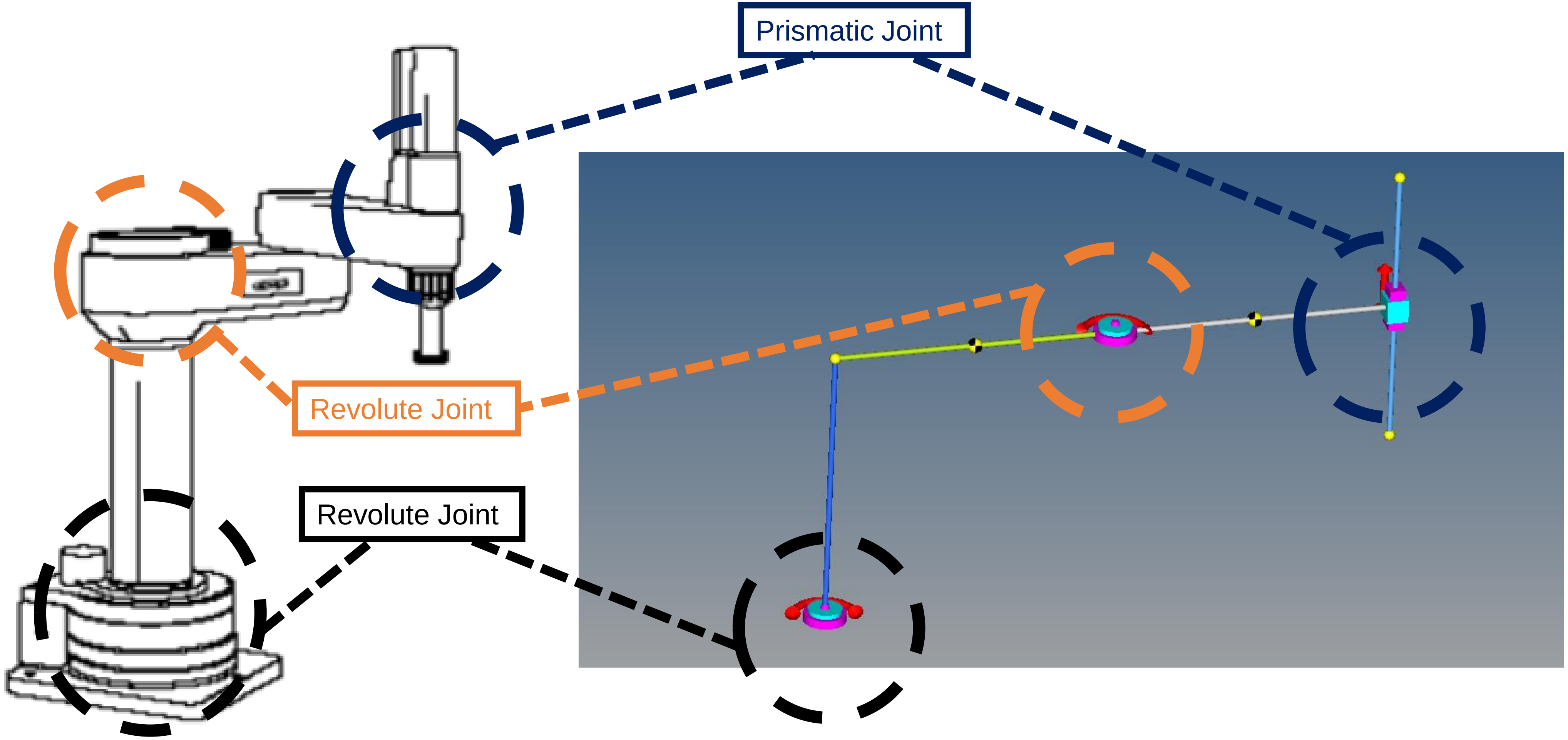


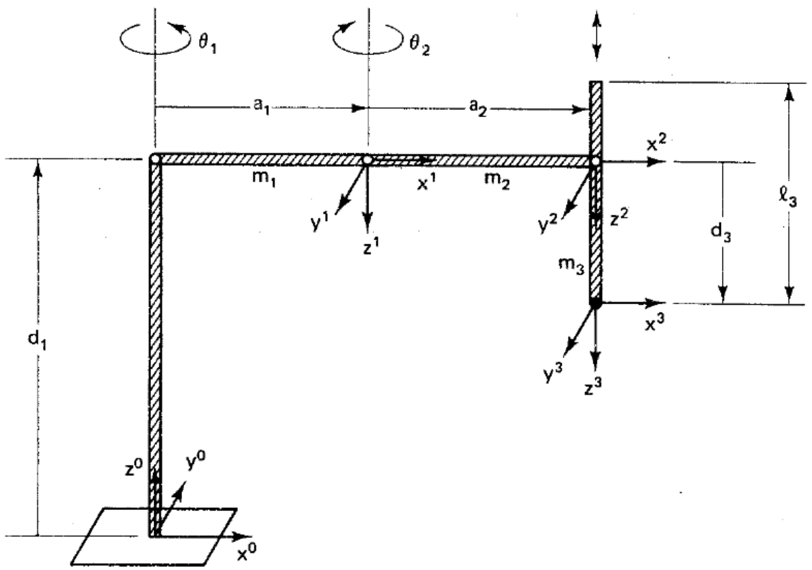
Fig. (1): 3-axis SCARA Manipulator [1]

Fig. (2): Simplified Model of 3-axis SCARA Manipulator [2]

Final analytical expression for joint torques [1]

$$\tau_1 = \left[\left(\frac{m_1}{3} + m_2 + m_3\right) a_1^2 + (m_2 + 2m_3) a_1 a_2 C_2 + \left(\frac{m_2}{3} + m_3\right) a_2^2\right] \ddot{q}_1$$
$$- \left[\left(\frac{m_2}{2} + m_3\right) a_1 a_2 C_2 + \left(\frac{m_2}{3} + m_3\right) a_2^2\right] \ddot{q}_2^2 + b_1(\dot{q}_1)$$
$$- a_1 a_2 S_2 \left[(m_2 + 2m_3)\dot{q}_1 \dot{q}_2 - \left(\frac{m_2}{2} + m_3\right) \dot{q}_2^2\right]$$

$$\tau_2 = -\left[\left(\frac{m_2}{2} + m_3\right) a_1 a_2 C_2 + \left(\frac{m_2}{3} + m_3\right) a_2^2\right] \ddot{q}_1 + \left(\frac{m_2}{3} + m_3\right) a_2^2 \ddot{q}_2$$
$$+ \left(\frac{m_2}{2} + m_3\right) a_1 a_2 S_2 \dot{q}_1^2 + b_2(\dot{q}_2)$$
$$\tau_3 = m_3 \ddot{q}_3 - g_0 m_3 + b_3(\dot{q}_3)$$



Dynamic Parameters

Link Masses	Link Lengths	Joint Frictions
$m_1 = 1\,kg$ $m_2 = 1\,kg$ $m_3 = 0.5\,kg$	$a_1 = 1\,m$ $a_2 = 1\,m$ $a_3 = 1\,m$	$b_1 = 0$ $b_2 = 0$ $b_3 = 0$

Generalized coordinates at given configuration of manipulator	Generalized velocities at given configuration of manipulator
$q_1 = 0\,rad$ $q_2 = 0\,rad$ $q_3 = 0.5\,m$	$\dot{q}_1 = 0.5\,rad/sec$ $\dot{q}_2 = 0.25\,rad/sec$ $\dot{q}_3 = 0.125\,m/sec$

Generalized accelerations at given configuration of manipulator	Moment of Inertias of links (links are thin cylinders and zero cross inertias)
$\ddot{q}_1 = 1\,rad/sec^2$ $\ddot{q}_2 = 1\,rad/sec^2$ $\ddot{q}_3 = 1\,m/sec^2$	$I_{xx} = 0$ for link 1 & 2 $I_{zz} = 0$ for link 3 $I_{yy} = ma^2/3$

Hard Code for 3 axis SCARA (based on analytical expressions)

```
%torque analytical equations
torque_1 = (((1/3)*m1 + m2 + m3)*(a1^2) + (m2 + 2*m3)*a1*a2*cos(theta_2) + .....
((1/3)*m2 + m3)*(a2^2))*theta_dot_dot1 - (((1/2)*m2 + m3)*a1*a2*cos(theta_2) + ....
((1/3)*m2 + m3)*(a2^2))*(theta_dot_dot2^2) - ....
a1*a2*sin(theta_1)*((m2 + 2*m3)*theta_dot1*theta_dot2 - (((1/2)*m2 +
m3)*(theta_dot2^2)))


torque_2 = -(((1/2)*m2 + m3)*a1*a2*cos(theta_2) + ((1/3)*m2 +
m3)*(a2^2))*theta_dot_dot1 + ....
((1/3)*m2 + m3)*(a2^2)*theta_dot_dot2 + ((1/2)*m2 +
m3)*a1*a2*sin(theta_2)*(theta_dot1)^2


torque_3 = m3*q_dot_dot3 - g*m3
```

Hard Code Output

```
torque_1 =

        2.83


torque_2 =

       -1.00


torque_3 =

       -4.41
```

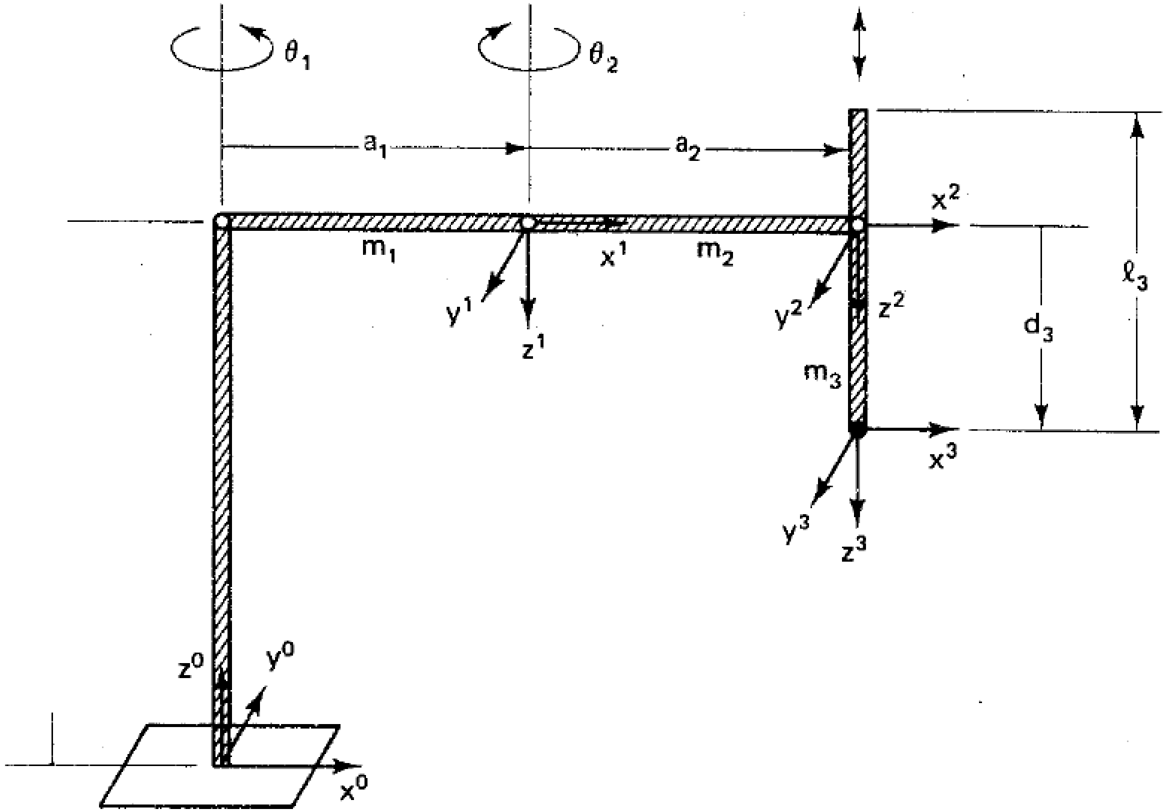
Torque applied at joint 1 in this configuration = 2.83 N-m
Torque applied at joint 2 in this configuration = -1 N-m
Force applied at joint 3 in this configuration = -4.41 N

Distal DH parameters for 3 axis SCARA

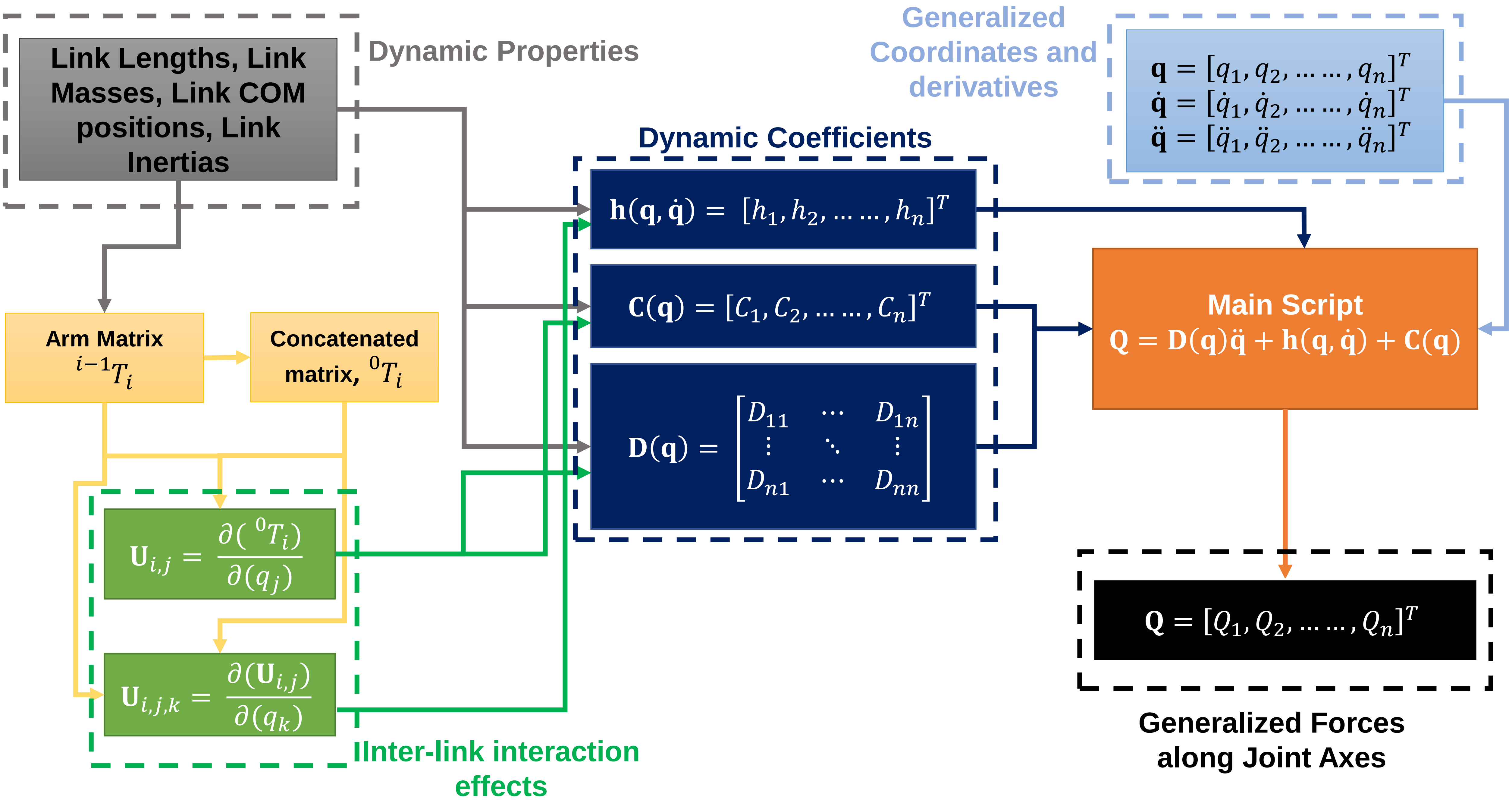
Frame(i)	θ_i	d_i	a_i	α_i	Home
1	θ_1	d_1	a_1	180	0
2	θ_2	0	a_2	0	0
3	0	q_3	0	0	$a_3/2$

Reference text file for dynamics codes

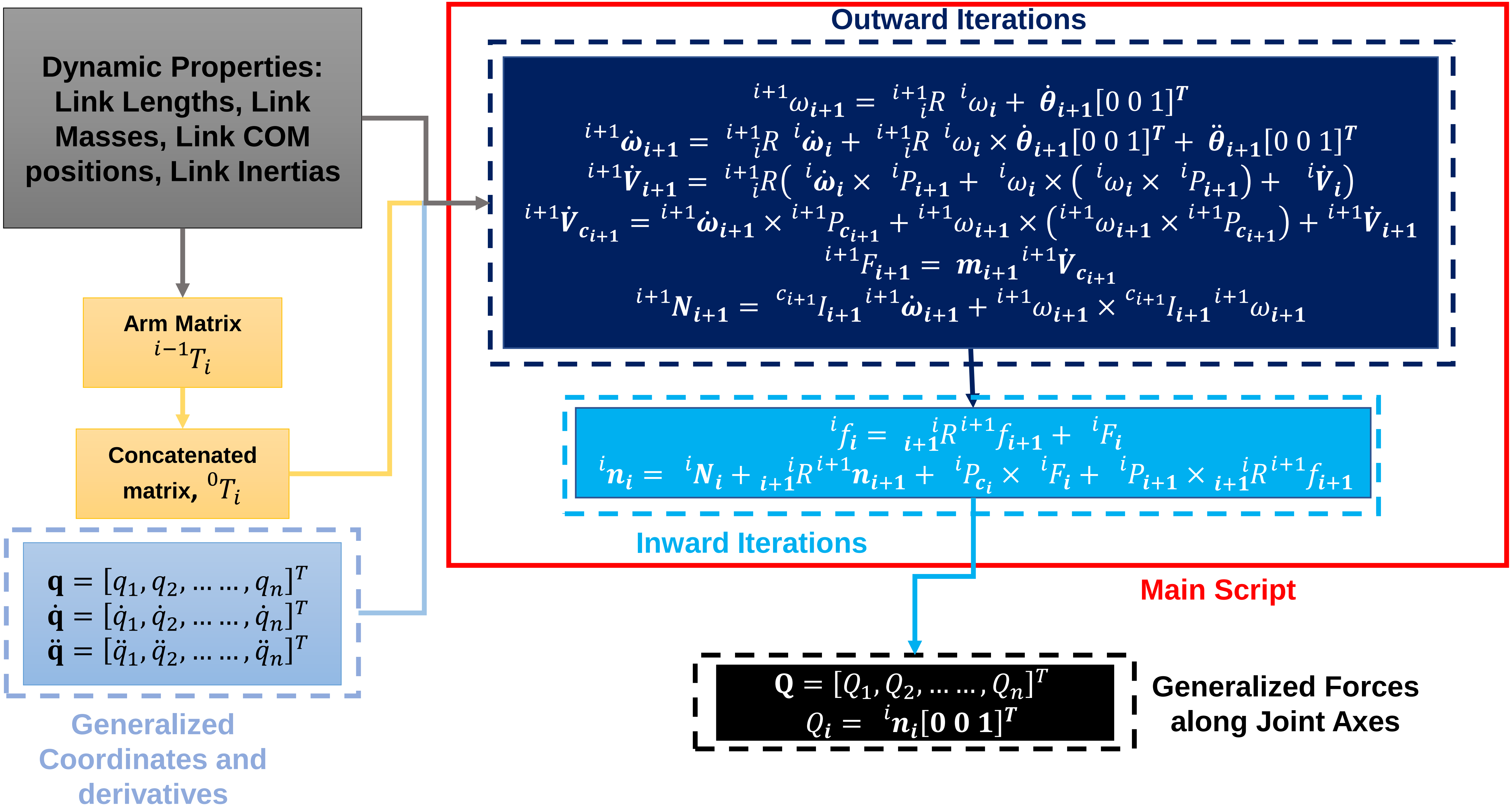
```
3
3

  180        1      1      0     0.5       1
   0         1      0      0     0.25      1
   0         0      0.5    0     0.125     0

% alpha(i)   a(i)       d(i)    theta(i)      dQ_dt(i)    flag(i)
   (deg)      (m)        (m)     (deg)         (rad/s)     (1/0)

%****************************************************************************
% POINTS TO BE NOTED
%****************************************************************************
% Line 1 = Number of Joints (NJ)
% Line 2 = Degree of Freedom of Cartesian Space (DOF)
```



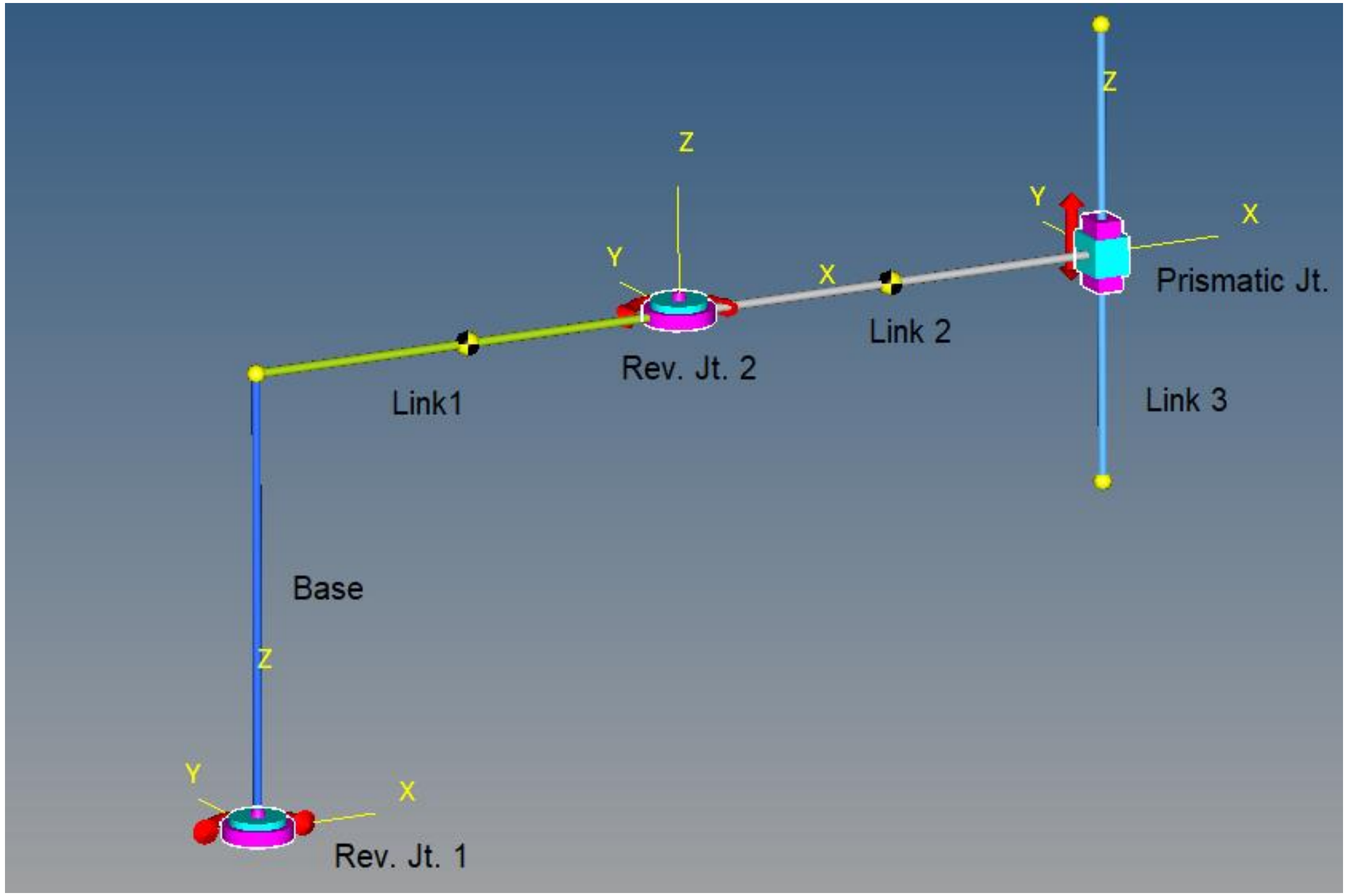
Dynamic Code Structure (Euler Lagrange) [3]



Dynamic Code Structure (Recursive Newton Euler Algorithm) [4]



Multi-Body Dynamics Simulation Setup



Motions defined for joints in model:

Revolute joint 1 ($q_1 = 0\, rad$, $\dot{q}_1 = 0.5\, rad/sec$, $\ddot{q}_1 = 1\, rad/sec^2$): $\dot{q}_1 = (1t + 0.5)\, rad/sec$

Revolute joint 2 ($q_2 = 0\, rad$, $\dot{q}_2 = 0.25\, rad/sec$, $\ddot{q}_2 = 1\, rad/sec^2$): $\dot{q}_2 = (1t + 0.25)\, rad/sec$

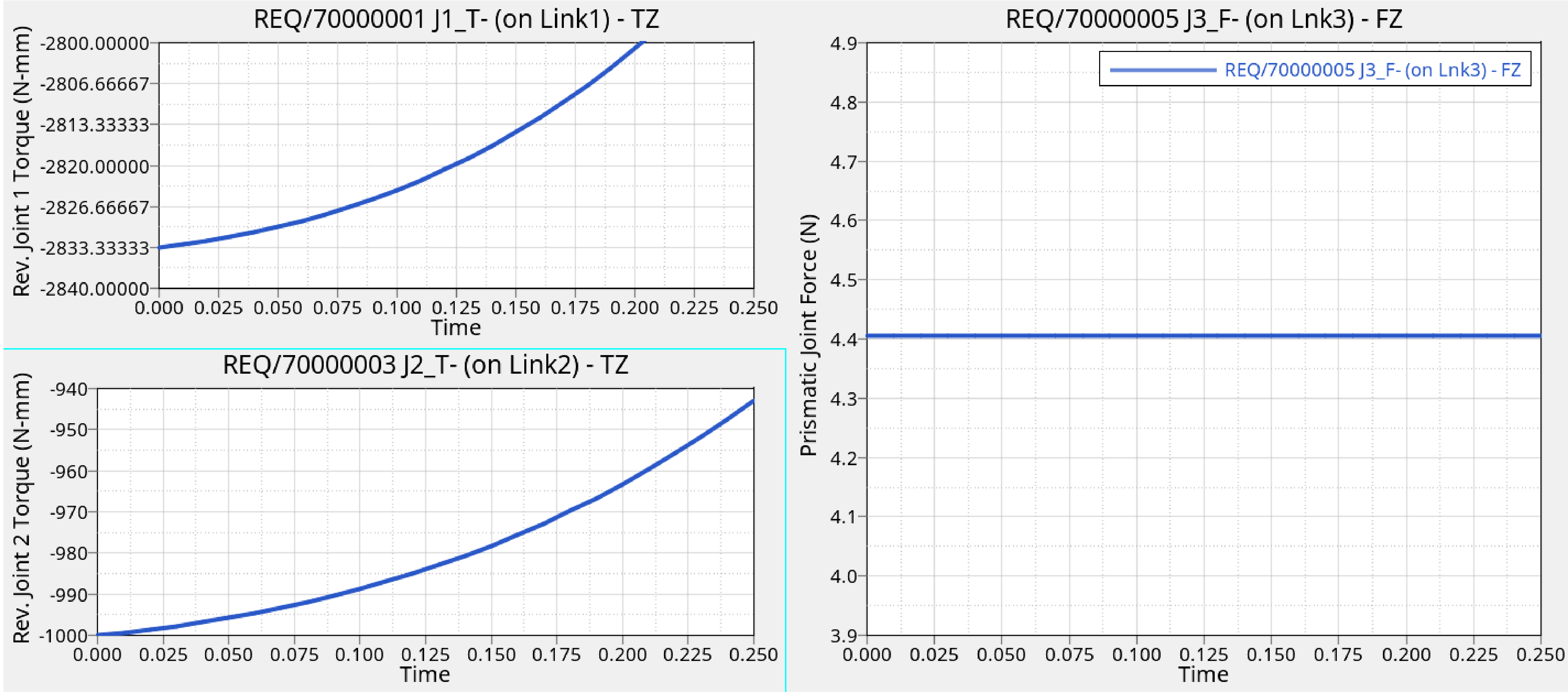
Prismatic Joint($q_3 = 0.5\, m$, $\dot{q}_3 = 125\, mm/sec$, $\ddot{q}_3 = 1000\, mm/sec^2$): $\dot{q}_3 = (1000t + 125)\, m/sec$

The simulation is run for 0.25 seconds

Code Result

Both the algorithms give the same result as the analytical equations.

MBD Simulation Result



The simulation at $t = 0$ gives the same result as the analytical solution

MBD Simulation Result

[1] Robert J. Schilling, ‘Manipulator Dynamics’ in *Fundamentals of Robotics: Analysis and Control,* 5th ed. New Delhi, India: Prentice Hall Inc., 2003, ch.6

[2] Altair Hyperworks Motionview (https://altairhyperworks.com/product/motionsolve/motionview), Last accessed on 10/02/2021

[3]. J. J. Uicker, "On the dynamic analysis of spatial linkages using 4 x 4 matrices," Ph.D. dissertation, Northwestern Univ., Aug. 1965.

[4]. J.Y.S Luh, M.W. Walker and R.P.C. Paul, “On-Line Computational Scheme for Mechanical Manipulators”, *Journal of Dynamic Systems, Measurement and Control,* Vol. 102, No. 6, pp. 69 76, 1980